

Term premiums and the predictability of recessions

Arturo Estrella* and Hao Wu**

February 2009

JEL Codes: G12, C53

Keywords: term structure, business cycle forecasting

Abstract: This paper examines systematically the role of expectations and term premium components of yields in forecasting recessions. We estimate a canonical term structure model that has been found in the earlier literature to have good properties, applying a new computationally efficient maximum likelihood technique that avoids ad hoc assumptions about estimation errors for particular maturities. Term premium estimates are subjected to validation tests. In contrast to earlier work, results suggest that term premiums may contain some information about future real economic activity but that decomposing nominal yields into expectations and term premium components does not significantly enhance predictive power.

* Corresponding author: Rensselaer Polytechnic Institute, SA 3rd Fl., Troy, NY 12180; tel. 1-518-276-2049; fax 1-518-276-2235; email: estrea@rpi.edu. ** Bendheim Center for Finance, Princeton University; email: hzwu@princeton.edu. The authors are grateful for comments and suggestions from Tobias Adrian, Emanuel Mönch, Hans Dewachter, seminar participants at the Federal Reserve Bank of New York, and participants at the November 2008 Banque de France conference on Financial Markets and Real Activity.

1. Introduction

Some researchers and policymakers have claimed in recent times that term premiums confound the ability of term spreads to forecast real activity and, in particular, recessions. Such claims are difficult to assess because of the variety of specific models for calculating term premiums and the absence of direct measures of accuracy for term premium estimates.

This paper examines systematically the role of expectations and term premium components of yields in forecasting future recessions. First, we use a canonical term structure model that has been found in the earlier literature to have good properties for decomposing yields into expectations and term premiums. Second, the model is estimated using a new computationally efficient maximum likelihood technique that avoids ad hoc assumptions about estimation errors for particular maturities. Third, term premium estimates are subjected to validation tests. Fourth, the expectations and term premium components of yields are used separately and jointly in probit models that have been shown earlier to be accurate predictors of recessions.

The empirical section of the paper uses U.S. Treasury yield data since 1959 from two sources: Fama-Bliss zero coupon rates from CRSP and zero coupon rates from the Federal Reserve Board. These yields are supplemented with shorter maturity Treasury bill data. We find that the characteristics of the input zero-coupon data may have substantive effects on the results, an aspect on which earlier literature has not focused. In contrast to some earlier work, results suggest that term premiums may contain useful information about future real economic activity not included in pure expectations, but that decomposing nominal yields into expectations and term premium components does not in general enhance predictive power.

2. Forecasting recessions with the term spread

2.1 Nominal term spreads

We start by providing a brief review of the predictive models that use a term spread to forecast recessions. Estrella and Hardouvelis (1991) introduced a simple probit model that uses the spread between 10-year and 3-month Treasury yields to predict recessions. The model has been quite successful in real time since then, and it has been shown to be stable even across changes in monetary policy regime.¹ The model is

$$R_{t+12} = \Phi(\alpha + \beta s_t) \quad (1)$$

where $R_t = 1$ if month t is labeled as a recession by the NBER and $R_t = 0$ otherwise, Φ is the normal cumulative distribution function, and $s_t = y_t^{(n)} - y_t^{(m)}$ is the spread between Treasury yields of long and short maturities. For instance, $n = 120$ and $m = 3$ months.²

Since the performance of the model has been well-documented in the literature, we focus here only on estimates of the equation for sample periods from 1960 to 2005 (Table 4) and 1972 to 2005 (Table 5), which best suit our purposes. The first sample period is consistent with standard estimates of the probit prediction model, whereas the second is chosen to conform with availability of zero-coupon data from the Federal Reserve Board, which we use in some of our term structure models. In both cases, the sample ends at the end of 2005 to avoid misclassifying observations of the recession dummy variable: the start of a recession depends on NBER dating of a cyclical peak, which is usually announced only after a long lag.

¹ See, e.g., Estrella, Rodrigues, and Schich (2003).

² See Estrella and Trubin (2006) for a recent discussion of the model, its estimation, and its performance.

Results in Tables 4 and 5 (first column) show that the term spread is highly significant in both estimation periods, even after standard errors are adjusted for heteroskedasticity and autocorrelation. The pseudo R^2 s are 25.1% and 31.3%, respectively.

2.2 Expectations and term premiums

A zero-coupon Treasury yield of maturity n is generally considered to consist of two components: one determined by expectations of future single-period yields, and another representing a premium that risk-averse investors require in order to hold the asset over a longer maturity. Symbolically,

$$y_t^{(n)} = e_t^{(n)} + p_t^{(n)} \quad (2)$$

where $e_t^{(n)} = \frac{1}{n} E_t \sum_{i=0}^{n-1} y_{t+i}^{(1)}$ and $p_t^{(n)} = y_t^{(n)} - e_t^{(n)}$. The “pure expectations hypothesis” of the term structure says that $y_t^{(n)} = e_t^{(n)}$ and thus $p_t^{(n)} = 0$. However, most empirical research has found evidence consistent with the assumption that $p_t^{(n)}$ is non-zero and time-varying.³

Following the above definitions, we decompose the term spread as

$$s_t = e_t^{(n)} - e_t^{(m)} + p_t^{(n)} - p_t^{(m)} \equiv e_t + p_t. \quad (3)$$

We may then ask: is the predictive power of the term spread derived from the expectations term, the term premium, or both? For instance, if the expectations term were dominant, inclusion of the term premium would only add noise and we may be better off by excluding it from the predictive model altogether. In order to gauge the importance of each term, we can estimate equations with each term individually,

$$R_{t+12} = \Phi(\alpha + \beta_1 e_t) \text{ and} \quad (4)$$

³ See, e.g., the summary in Campbell (1995).

$$R_{t+12} = \Phi(\alpha + \beta_2 p_t), \quad (5)$$

or include both terms and let the data determine the appropriate weights:

$$R_{t+12} = \Phi(\alpha + \beta_1 e_t + \beta_2 p_t). \quad (6)$$

Note that the model in equation (1) imposes the restriction $\beta_1 = \beta_2$ in equation (6), whereas (4) and (5) correspond to $\beta_2 = 0$ and $\beta_1 = 0$, respectively.

A few researchers have looked into this question recently, and even some influential policymakers have weighed in with their own views. Among the latter, Fed Chairman Greenspan said in November 2005 that

Although the slope of the yield curve remains an important financial indicator, it needs to be interpreted carefully. In particular, a flattening of the yield curve is not a foolproof indicator of future economic weakness. ... The connection between future output growth and the other two factors affecting the slope of the yield curve – the gap between near-term and long-term inflation expectations and the difference between near-term and long-term risk premiums – is far less certain and likely to depend on economic circumstances.⁴

More recently, Chairman Bernanke, echoed the same thoughts in March 2006:

I would not interpret the currently very flat yield curve as indicating a significant economic slowdown to come, for several reasons. ... to the extent that the flattening or inversion of the yield curve is the result of a smaller term premium, the implications for future economic activity are positive rather than negative.⁵

⁴ Responses submitted by Chairman Greenspan to written questions received from Chairman Saxton in connection with the Joint Economic Committee hearing on November 3, 2005.

⁵ Remarks by Chairman Ben S. Bernanke before the Economic Club of New York, March 20, 2006.

On the research side, Wright (2006) found that

While a probit model using the term spread alone predicts high odds of a recession in the next four quarters, the other probit models that I estimate, which all control for the level of the funds rate, do not. This gives formal empirical support to a view that has been widely expressed by commentators that the present flatness of the yield curve is a reflection of low term premiums rather than especially tight monetary policy, and this flatness accordingly does not seem to herald a sharp slowdown.⁶

The evidence in Wright (2006) is indirect in that the paper does not include an explicit measure of term premiums in any of the equations.

Subsequently, however, Rosenberg and Maurer (2008) estimated probit models of the form (4)-(6), where they based term premium estimates on results of a term structure model developed by Kim and Wright (2005).⁷ Rosenberg and Maurer conclude that

Our empirical analysis finds that the expectations component is indeed a leading indicator of recession, while the term premium component is not. When we compare the historical recession forecasting performance of the term spread and its expectations component, we find some evidence that a model based on the expectations component is more accurate. Our results should be interpreted with

⁶ Jonathan H. Wright, "The Yield Curve and Predicting Recessions" Finance and Economics Discussion Series No. 2006-07, Federal Reserve Board, February 2006.

⁷ Updated results for the Kim-Wright are provided at the Federal Reserve Board web site, <http://www.federalreserve.gov/pubs/feds/2005/200533/FEDS200533.xls>.

caution, however, because they rely on imprecise estimates of the term premium and on a sample period that includes only six recessions.⁸

The last sentence points to a critical issue in this literature. Term premiums are never observed directly, but are estimated from elaborate term structure models that impose a number of strong identifying assumptions. What degree of confidence can we have in the accuracy of the resulting term structure estimates?

In the next section, we construct a particular term structure model that exhibits characteristics that have been found in the literature to be helpful in producing accurate estimates of expected short-term rates. The model is then estimated and subjected to indirect tests of accuracy of expected rates of different maturities and, by implication, of the corresponding term premiums. Only after the model has been validated in these ways do we proceed with the estimation of the probit prediction models and the associated statistical inference.

3. Term structure model and estimation method

3.1 Form of the model

The basic model is a discrete-time essentially-affine no-arbitrage term structure model. The errors in the state transition equation are homoskedastic, so in the Duffee (2002) nomenclature the model is of the $A_0(q)$ type. The choice of this formulation is based on earlier empirical tests against various heteroskedastic alternatives $A_v(q)$, which allow the covariance of the stochastic process to depend on v of the q factors. Duffee (2002) finds that the homoskedastic model produces the most accurate forecasts of future yields, and Dai and

⁸ Joshua V. Rosenberg and Samuel Maurer, "Signal or Noise? Implications of the Term Premium for Recession Forecasting" Federal Reserve Bank of New York Economic Policy Review, Forthcoming.

Singleton (2002) show that expectations based on the homoskedastic model performs best in Campbell-Shiller tests of the expectations hypothesis (discussed in Section 4.3).

Suppose a term structure model contains observed zero-coupon yields for n maturities, which are modeled as affine functions of q state variables (or latent factors) with $q < n$. The observation equation is

$$y_t = a + b'x_t + u_t, \quad (7)$$

where y_t is a vector of observed yields for n different maturities, expressed as decimal return per period, x_t is a vector of q state variables, $q < n$, and a and b are conformable constant matrices. The errors u_t are iid with normal distribution $N(0, R)$.

Following, for instance, Ang and Piazzesi (2003) the dynamics of the state variables are modeled in discrete time. Dai, Le and Singleton (2006) show that this type of discrete-time model converges to a Duffie-Kan (1996) continuous-time model as the length of the time period goes to zero.⁹ The transition equation for x_t is a VAR(1),

$$x_t = \delta + Mx_{t-1} + \varepsilon_t, \quad (8)$$

where the errors ε_t are iid with normal distribution $N(0, Q)$ and are orthogonal to the u_t . The implication is that x_t is normally distributed unconditionally as $N((I - M)^{-1} \delta, V_x)$ where V_x is the unique solution to $V_x = MV_xM' + Q$.

The assumption that there are no arbitrage opportunities across bonds of different maturities imposes restrictions on the parameters of equation (7). In the discrete-time model framework, suppose that the price of risk at time t is given by the affine function

⁹ Note, however, that the continuous model integrated to match the discrete time period is not the same as the discrete model. See Appendix 2.

$$\Lambda_t = \lambda_0 + \lambda_1 x_t. \quad (9)$$

For maturities of $m = 1, 2, \dots$ periods, define the sequences of scalars $\{A_m\}$ and of q -dimensional vectors $\{B_m\}$ by the recursive relationships

$$B_m = (M - \Sigma \lambda_1)' B_{m-1} + B_1 \text{ and} \quad (10)$$

$$A_m = A_{m-1} + (\delta - \Sigma \lambda_0) B_{m-1} - \frac{1}{2} B_{m-1}' Q B_{m-1} + A_1, \quad (11)$$

where $A_0 = 0$ and $B_0 = 0$ and Σ is such that $Q = \Sigma \Sigma'$, say the Choleski decomposition. If the maturity of the i th element of y_t is the integer $m(i)$, the no-arbitrage assumption requires that¹⁰

$$a_i = A_{m(i)} / m(i) \text{ and} \quad (12)$$

$$b_i = B_{m(i)} / m(i). \quad (13)$$

Note that if the yields y_t are rescaled by a factor γ , say they are expressed as percent per annum so that $\gamma = 1200$, the quadratic term in B_{m-1} in (11) must be multiplied by γ^{-1} . Also note that δ and A_1 are not separately identifiable, so we follow Ang and Piazzesi (2003) and take $\delta = 0$.

Given estimates of the latent factors x_t , the free parameters in

$\Theta = \{M, A_1, B_1, \lambda_0, \lambda_1, Q, R\}$ may be estimated by maximum likelihood, based on the conditional distribution of $y_t \mid y_{t-1}$ for each observation $t = 1, \dots, T$. Under the assumptions of the model, this distribution is $N(\mu_t, \Omega_t)$ with $\mu_t = a + b' E(x_t \mid y_{t-1})$ and $\Omega_t = b' V(x_t \mid y_{t-1}) b + R$. The log likelihood function, up to a constant, is

$$L(\Theta) = -\sum_{t=1}^T (\log |\Omega_t| + (y_t - \mu_t)' \Omega_t^{-1} (y_t - \mu_t)). \quad (14)$$

¹⁰ See, for example, Ang and Piazzesi (2003) for a derivation of the recursive formulas in the discrete-time model.

Affine ($\lambda_1 = 0$) or essentially affine ($\lambda_1 \neq 0$) term structure models are generally estimated under one of two assumptions with regard to the variances of the n observation errors u_t . Specifically, these variances may be unrestricted or some of them may be assumed to be zero. A summary of these approaches is given in Appendix 1. The following section describes the method used here, which is computationally efficient and avoids ad hoc assumptions about individual error variances.

3.2 Estimation by exact principal factors

The method of the present paper assumes that yields of all maturities are observed with error, but that the covariance matrix of the errors is scalar. This assumption implies that the q state variables or factors are then exactly identified and computable by linear algebra without the need for the Kalman filter or ad hoc assumptions. Moreover, in contrast to the Chen-Scott approach (see Appendix 2), direct estimation of multiple observational variances is unnecessary, leading to additional computational speed.

Thus, let $R = \sigma^2 I$. Tipping and Bishop (1999) show that this assumption is sufficient to allow for the direct calculation of maximum likelihood estimates of equation (7) and of the conditional expectations $E(x_t | y_t)$ by linear algebra, without the need for a Kalman filter. We exploit this result in two ways. First, if no-arbitrage constraints are not imposed, the method produces direct maximum likelihood estimates of the latent factors and of the model parameters. Second, the fact that q principal components of the yields are fitted without error by this method allows for a straightforward extension to the case with no-arbitrage constraints.

The assumption that R is scalar treats all errors symmetrically. In models with three or more factors, all errors tend to be close to zero and the constraint thus holds at least as a

numerical approximation. Even if the constraint were to be formally rejected, the numerical estimates may be very useful as initial values for estimation of the general model with the Kalman filter, which is sensitive to the starting point.

The solution is based on an eigendecomposition of, V , the unconditional covariance matrix of y_t , given by $V = b'V_x b + R$. Let

$$V = UDU', \quad (15)$$

where U is the orthogonal matrix whose columns are the eigenvectors of V , and D is the diagonal matrix of eigenvalues, ordered from large to small. Partition these matrices as $U = [U_q \ U_{-q}]$

and

$$D = \begin{bmatrix} D_q & 0 \\ 0 & D_{-q} \end{bmatrix}, \quad (16)$$

where U_q contains the first q columns of U , U_{-q} contains the last $n - q$ columns of U , D_q is $q \times q$, etc. For simplicity and without loss of generality, assume for the moment that $V_x = I$, as in Tipping and Bishop (1999). This assumption does not affect the estimates of y_t or the fit of the model. Tipping and Bishop (1999) show that the maximum likelihood estimates are

$$\sigma^2 = \text{tr}(D_{-q}) / (n - q) \quad (17)$$

$$a = \bar{y} = 1/T \sum y_t \quad (18)$$

$$b = (D_q - \sigma^2 I)^{1/2} U_q' \quad (19)$$

$$x_t = D_q^{-1} (D_q - \sigma^2 I)^{1/2} U_q' (y_t - \bar{y}). \quad (20)$$

The fitted values of y_t are given by

$$\hat{y}_t = a + b'x_t = \bar{y} + (U_q U_q' - \sigma^2 U_q D_q^{-1} U_q') (y_t - \bar{y}). \quad (21)$$

The above estimates, which do not impose the no-arbitrage constraints, are sufficient for some purposes but not for others. For example, in addition to fitted contemporaneous values of the yields included in y_t , the estimated model may be used to forecast future yields of the same maturities. Specifically, equation (8) may be estimated using the generated x_t and used to forecast future values of the state variables using time t information. These expectations in turn may be substituted in equation (7) to forecast future bond yields. However, if the no-arbitrage constraints hold, imposing them in estimation would produce more efficient estimates. Moreover, with no-arbitrage constraints, the model could be used to predict rates of all maturities, not just those included in y_t .

To estimate the model under the no-arbitrage constraints, we use an implication of the maximum likelihood solution: that the first q principal components of y_t are fitted exactly by the maximum likelihood estimates. To see this, define $y_t^{(q)} = U_q' y_t$, the vector of the first q principal components of y_t and consider the results of applying the estimation method to $y_t^{(q)}$.

The variance of $y_t^{(q)}$ is $U_q' V U_q = D_q$. This matrix is given directly in eigendecomposition form, with the eigenvector matrix being I_q , the $q \times q$ identity matrix.

Moreover, no principal components of $y_t^{(q)}$ are excluded, so that $\sigma^2 = 0$ in (17) and, from (21),

$$\hat{y}_t^{(q)} = \bar{y}^{(q)} + I_q I_q' (y_t^{(q)} - \bar{y}^{(q)}) = y_t^{(q)}. \quad (22)$$

Thus, the principal component vector $y_t^{(q)}$ is fit exactly, making it possible to compute the factors x_t in terms of a , b and y_t by the exact linear equation

$$y_t^{(q)} = U_q' a + U_q' b' x_t, \quad (23)$$

which may be inverted to obtain

$$x_t = (U'_q b')^{-1} (U'_q y_t - U'_q a). \quad (24)$$

This step is analogous to the method Chen and Scott (1993) use to compute exact factors. See equation (36) in Appendix 2, which summarizes the Chen-Scott method.

To estimate the no-arbitrage model by maximum likelihood, we apply equations (12), (13) and (24). The likelihood function (14) is computed using $E(x_t | y_{t-1}) = Mx_{t-1}$,

$V(x_t | y_{t-1}) = Q$, and $E\varepsilon_t u'_t = 0$ for $t = 2, \dots, T$. Thus,

$$L(\Theta) = -\sum_{t=2}^T (2\log \|bU_q\| + \log|Q| + \varepsilon'_t Q^{-1} \varepsilon_t + \log|R| + u'_t R^{-1} u_t), \quad (25)$$

where $\varepsilon_t = x_t - Mx_{t-1}$ and $u_t = y_t - a - b'x_t$, evaluated at the estimated values, and $\| \cdot \|$ denotes the absolute value of the determinant. For $t = 1$, one may again suppose that the unknown x_0 follows its unconditional distribution and that $E(x_1 | y_0) = 0$ and $V(x_1 | y_0) = V_x$.

4. Empirical estimates of term structure model

4.1 Data

For the empirical estimates, we use zero-coupon data from two alternative sources: the Center for Research on Securities Prices (CRSP) and the Board of Governors of the Federal Reserve. There is some earlier evidence that the method used to compute zero-coupon yields may affect term structure model estimates,¹¹ and we find some quantitative and qualitative differences in the estimates derived from the two data sets. We treat the data from CRSP as the base case, but for completeness we also report results derived from Board data.

From CRSP, we use Fama-Bliss zero-coupon yields of maturities 1, 3, 12, 24, 36, 48 and 60 months from January 1959 to December 2007. The bond yields, corresponding to maturities

¹¹ See Dai, Singleton and Wei (2004) and Singleton (2006, Section 9.6).

12 months and higher, are from the CRSP “Fama zero-coupon” files, whereas the short-term yields, for 1 and 3 months, are from the CRSP “Fama Treasury Bill” files. We refer to this data set as “Fama-Bliss data.”

Board zero-coupon data are derived from the off-the-run term structure of nominal Treasury monthly yields of maturities 1 through 10 years, as computed by Gökaynak, Sack, and Wright (2006). Bond yields for maturities from 1 to 7 years are available from June 1961 to December 2007, and bond yields from 8 to 10 years are available from August 1971 to December 2007. For short-term rates, we use secondary market 3-month and 6-month Treasury bill rates from June 1961 to December 2007, expressed on a bond-equivalent basis as described in Estrella and Trubin (2006). We refer to this data set as “Board data.” Table 1 presents some sample statistics for both data sets.

4.2 Empirical results

We first apply the three computational approaches (exact principal factors, Chen-Scott, Kalman filter) to the essentially affine model of U.S. Treasury term structure rates using Fama-Bliss data since 1959. Our reported results are based on a three-factor model, which is the norm in the literature. In our base case, the three factors account for over 99% of the variance in the data. We also estimated a model with four factors, but found no evidence that the fourth factor affected results in any substantive way.

Table 2 contains parameter estimates and their respective standard errors. Note that, in contrast to most of the earlier literature, we estimate all the identifiable parameters of the model. Because of the large number of parameters, it is all but inevitable that many of them will not differ from zero at standard levels of statistical significance. It is common practice in the

literature to set the insignificant parameters to zero and estimate only the others, which typically results in higher significance levels for the remaining parameters. However, the zero restrictions are ad hoc and vary from application to application. Since many of the point estimates of key parameters are economically significant, we prefer to estimate all parameters and report results without imposing ad hoc zero restrictions.

Aside from the covariance matrices of residuals, the parameters of the model may be classified into three categories: the coefficient matrix M of the transition equation, the price of risk parameters λ , and the affine function coefficients δ in the equation for the one period rate. The estimates show the matrix M to be nearly diagonal, with statistically significant diagonal elements. Two of the diagonal elements are close to 1, implying that the associated state variables are very persistent, and the third value is about $2/3$. The state variables are closely related to the first three principal components of the yields and to the level, slope and curvature factors as in Litterman, Scheinkman and Weiss (1991), though they are not the same.¹²

The price of risk parameters are relatively numerous and less precisely estimated than the others, which is consistent with earlier research. In general, they are not statistically significant, but several of the parameters are large enough to be economically significant (e.g., λ_{01} , λ_{03} , λ_{13} , λ_{31} , λ_{33}). The overall effect of these parameters is substantial, as indicated by the size of the time-varying risk premiums discussed below.

The constant coefficient δ_0 of the affine function equation for the one period rate is estimated with a large standard error, but the point estimate is very close to its conceptual

¹² In the case of estimation by exact principal factors, the state variables span the same space as the first three principal components by construction, but are not identical to them. In general, the application of no-arbitrage constraints introduces differences between the state variables and unconditional principal components.

equivalent, the unconditional average of the one-period rate. The coefficients δ_1 of the state variables in the one period rate equation are estimated with varying degrees of accuracy, but allow for substantial influence on the time series behavior of the short rate.

Table 2 also provides two measures of the fit of the model, the log likelihood and the mean standard error, which is more intuitive. The mean standard error is defined as

$$\frac{1}{nT} \sum_{i=1}^n \sum_{t=1}^T u_{it}^2 \quad (26)$$

where u_{it} is the i th element of u_t . With estimation by exact principal factors, the mean standard error is a direct estimate of the variance σ^2 . With other estimation methods which may allow for different σ_i^2 for each maturity, the mean standard error represents a weighted average of these individual variances. Error covariance estimates under each of the three methods are given in Table 3.¹³

Figure 1 shows the term premiums implied by the model using the base case Fama-Bliss data for maturities of 3 months and 5 years. Cross-sectionally, the term premiums tend to be higher for longer maturities and range from essentially zero at the one-month maturity to the values in the figure for the 5-year maturity.

To put the foregoing results in a broader context, we compare the term premiums with those obtained using a different data set (Board data as described above) as well as the results of the model by Kim and Wright (2005). The top panel of Figure 2 compares these values for a 5 year maturity in all three cases. The model using Board data is estimated using all maturities up to 5 years. The figure shows values from July 1990 to December 2007 to match the period of

¹³ Note that the covariance matrix in the Kalman filter case is restricted to be scalar, so that it corresponds to the base case estimates.

availability for the Kim-Wright data. The results for the three series are clearly similar though not identical. The correlation between the base case and the Board data is 98.6% and between the base case and the Kim-Wright model it is 96.3%.

The bottom panel of Figure 2 performs a similar comparison, but 10-year term premiums are shown for the Board and Kim-Wright models. In the case of the Board model, all maturities up to 10 years are used for estimation. A clear feature of this panel is that the 10-year premiums tend to be higher than the 5-year premiums, which is consistent with the internal results under the base case model. However, despite the difference in maturities, correlations are about the same or higher as with the 5-year term premiums. The correlation between the base case and the Board data is 98.4% and between the base case and the Kim-Wright model it is 99.4%.

4.3 Validation tests

We perform two types of validation tests on the base case results and compare the results with the application of the same tests with alternative data and with the Kim-Wright model. The first test uses the model assumption that the disturbances u_{it} for each maturity are serially uncorrelated and cross-sectionally uncorrelated. If this is the case, then the error $u_{mt} - u_{kt}$ in the term spread for any two maturities m and k should be serially uncorrelated as well. Figure 3 plots the autocorrelation function of spread errors for $m = 60$ and $k = 3$ months. When viewed in isolation, results for the base case using Fama-Bliss data are not too impressive, since the autocorrelations for 1 and 2 lags appear large and statistically significant. However, the base case does seem to improve on the other two cases in this respect, since the autocorrelation function for them is nearly uniformly higher across all lags. The differences at short lags in particular are quite large.

The second test is based on the so-called Campbell-Shiller tests of the expectations hypothesis. If the pure expectations hypothesis holds, long-term rates are weighted averages of future one-period rates. An implication of this hypothesis is that the one-period expected return on a bond of maturity m is related to the term spread between the m -month and 1-month rate in the following way:

$$E_t \left(y_{t+1}^{(m-1)} - y_t^{(m)} \right) = \frac{1}{m-1} \left(y_t^{(m)} - y_t^{(1)} \right). \quad (27)$$

Thus, a regression of the form

$$y_{t+1}^{(m-1)} - y_t^{(m)} = \alpha^{(m)} + \beta^{(m)} \frac{1}{m-1} \left(y_t^{(m)} - y_t^{(1)} \right) + \nu_t^{(m)} \quad (28)$$

should produce estimates $\alpha^{(m)} = 0$ and $\beta^{(m)} = 1$. Typical results in the literature show that these equations tend to produce significant estimates $\beta^{(m)} < 0$, especially for longer maturities, rejecting the pure expectations hypothesis.

However, term premiums from a term structure model may be used to construct a correction term for the dependent variable in the regression (28) that may result in estimates of $\beta^{(m)}$ close to 1, indirectly validating the accuracy of the estimated risk premiums. The adjusted regression is

$$y_{t+1}^{(m-1)} - y_t^{(m)} - C_t^{(m)} = \alpha^{(m)} + \beta^{(m)} \frac{1}{m-1} \left(y_t^{(m)} - y_t^{(1)} \right) + \nu_t^{(m)}, \quad (29)$$

where

$$C_t^{(m)} = p_{t+1}^{(m-1)} - \frac{m}{m-1} p_t^{(m)}. \quad (30)$$

Figure 4 shows estimates of $\beta^{(m)}$ for models using Fama-Bliss and Board data up to 5 years maturity. In each case, there is a curve representing results that include maturities under

one year and a curve that uses only 1 to 5 years in increments of 1 year. Statistically, none of the estimates are significantly different from 1, supporting the accuracy of the underlying term structure models. However, there are some qualitative differences in that results using the Fama-Bliss data are more robust and, in general, closer to 1. This pattern especially applies to the base case using all available Fama-Bliss maturities.

Figure 5 shows similar statistics for a range of models based on Board data. The various experiments look at the results of adding short-term rates, adding long-term rates, and changing the sample period. Results suggest that the largest effects are obtained with adding rates under 1 year, which is consistent with results in Figure 4. In all cases, longer maturities produce estimates of $\beta^{(m)} < 1$, though the difference is not statistically significant and the level is not negative, as tends to occur when unadjusted returns are used as dependent variables.

5. Application to recession forecasting

Having obtained estimates of the expectation and term premium components of yields of all maturities, we can calculate the term spreads that have been shown earlier to be good forecasters of recessions, decompose them into an expectations component and a term premium component, and examine whether the decomposition is helpful in forecasting recessions.

The earlier literature has identified the spread between 10-year and 3-month Treasury yields as having strong and robust predictive power for recessions one year ahead.¹⁴ For the alternative models based on Board data and Kim-Wright estimates, we use that combination of maturities in our predictive equations. For the base case model, however, we use the spread

¹⁴ See Estrella and Trubin (2006) for a discussion.

between 5-year and 3-month yields because 10-year Fama-Bliss data are not available. For each of the three cases, we estimate probit equations of the forms (1) and (4)-(6).¹⁵

Results for the base case model are provided in Table 4, which estimates the models with data from January 1960 to December 2005.¹⁶ The predictive variable in the first column is the spread used in Estrella and Hardouvelis (1991) and much of the subsequent literature, which is presented here for comparison with results using the present data sets. The second column shows that results using the 5-year 3-month base case spread are very similar to the traditional estimates in terms of both coefficient estimates and significance.¹⁷ The fit is a bit less close for the base case data, perhaps because a 10-year rate is not available. When the two components of the spread are used separately in columns (3) and (4), we see that only the expectation component is significant, as in Rosenberg and Maurer (2008).

The last column of Table 4 is perhaps the most interesting. When both components are included jointly, both are significant and the point estimates of the coefficients are relatively close.¹⁸ In fact, a Wald test for equality of the two coefficients has a p value of .349. The implication is that for the purposes of predicting recessions one year ahead, decomposing the spread into expectations and term premium components does not produce better results in terms

¹⁵ Equations (4)-(6) contain generated regressors, which may introduce inconsistency in the estimation of covariances of parameter estimates. To address this potential problem, we applied a two-step maximum likelihood estimator as in Murphy and Topel (1985) but found no appreciable effects on the covariance matrix estimates.

¹⁶ As noted earlier, we stop the estimation sample at December 2005 so as to avoid doubts about dating of recessions in more recent periods.

¹⁷ Significance is calculated from consistently estimated standard errors, using a Newey-West correction with 12 lags.

¹⁸ This result contrasts with the corresponding equation in Rosenber and Maurer (2008).

of statistical significance. Of course, having an accurate estimate of the term premium provides valuable information in and of itself, so the exercise is far from pointless.

Are these results robust to the use of the two alternative models? Tables 5 and 6, which are analogous in form to Table 4, suggest that the results are generally robust. Table 5 uses the model based on Board data. To include the 10-year rate as the long rate, we use a sample from August 1972 to December 2005. Qualitatively, the results of Table 5 are very similar to those of Table 4. Two slight differences are that the fit of the equation using Board data (column (2)) is a bit better than the benchmark (column (1)) and that the coefficients of the two components in column (5) are even closer than in Table 4 (p value of Wald test is .930).

Table 6 presents the same analysis using Kim-Wright estimates and the results are comparable. However, some results are different, possibly because of the very short estimation sample resulting from data availability. For instance, in this case the risk premium component is significant at standard levels both jointly and by itself. However, the coefficients of the two components in equation (5) are different here (p value is .009), suggesting that the decomposition is useful for predictive purposes. The short time period that includes only two recessions suggests that the results be taken with a grain of salt.¹⁹

6. Conclusions

The term structure model developed in this paper produces reasonable estimates of term premiums that are generally consistent with the earlier literature – particularly as to their level and time variation. Moreover, we use a computationally efficient estimation method and validation tests to enhance confidence in the new results. The decomposition of spreads into

¹⁹ Again see comments by Rosenberg and Maurer (2008).

expectations and term premium components leads to results that differ from the literature as to the role of term premiums in the prediction of future recessions. Here, we find little evidence that decomposing a term spread into expectation and term premium components helps forecast recessions.

References

- Ang, A., and Piazzesi, M. (2003), "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables," *Journal of Monetary Economics*, 50, 745-787.
- Campbell, J. Y. (1995), "Some Lessons from the Yield Curve," *Journal of Economic Perspectives*, 9, 129-52.
- Chen, R. R., and Scott, L. (1993), "Maximum Likelihood Estimation for a Multifactor Equilibrium Model of the Term Structure of Interest Rates," *Journal of Fixed Income* 3, 14-31.
- Dai, Q., Le, A., and Singleton, K. (2006), "Discrete-time Dynamic Term Structure Models with Generalized Market Prices of Risk," working paper.
- Dai, Q., and Singleton, K., (2000), "Specification Analysis of Affine Term Structure Models," *Journal of Finance* 55, 1943-1978.
- Dai, Q., and Singleton, K., (2002), "Expectations Puzzle, Time-Varying Risk Premia, and Affine Models of the Term Structure," *Journal of Financial Economics* 63, 415-441.
- Dai, Q., Singleton, K., and Wei, Y. (2004), "Regime Shifts in a Dynamic Term Structure Model of U.S. Treasury Bond Yields," *Proceedings*, Federal Reserve Bank of San Francisco, March.

- Dejong, F. (2000), "Time Series and Cross-Section Information in Affine Term-Structure Models," *Journal of Business and Economic Statistics*, 18, 300-314.
- Duffee, G. (2002), "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 57, 405-443.
- Duffie, D., and Kan, R. (1996), "A Yield-factor Model of Interest Rates," *Mathematical Finance*, 6, 379- 406.
- Estrella, A., and Hardouvelis, G. (1991), "The Term Structure as a Predictor of Real Economic Activity," *Journal of Finance*, 46, 555-576.
- Estrella, A., Rodrigues, A., and Schich, S. (2003), "How Stable Is the Predictive Power of the Yield Curve? Evidence from Germany and the United States," *Review of Economics and Statistics*, August 2003.
- Estrella, A., and Trubin, M. R. (2006), "The Yield Curve as a Leading Indicator: Some Practical Issues," *Current Issues in Economics and Finance*, Federal Reserve Bank of New York, July/August 2006.
- Fisher M. and Gilles C. (1996), "Estimating Exponential-Affine Models of the Term Structure," working paper.
- Gürkaynak, R. S., Sack, B. and Wright, J. H. (2006), "The U.S. Treasury Yield Curve: 1961 to the Present," *Finance and Economics Discussion* No. 2006-28, Federal Reserve Board, 2006.
- Kim, D., and Wright, J. H. (2005), "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates," *Finance and Economics Discussion* No. 2005-33, Federal Reserve Board, 2005.

- Litterman, R., Scheinkman, J., and Weiss, L. (1991), “Volatility and the Yield Curve,” *Journal of Fixed Income*, 1, 49–53.
- Murphy, K., and Topel, R. (1985), “Estimation and Inference in Two-Step Econometric Models,” *Journal of Business and Economic Statistics* 3, 370-379.
- Rosenberg, J. V., and Maurer, S. (2008), “Signal or Noise? Implications of the Term Premium for Recession Forecasting,” *Economic Policy Review*, Federal Reserve Bank of New York, July.
- Singleton, K. (2006), “Empirical Dynamic Asset Pricing: Model Specification and Econometric Assessment,” Princeton University Press.
- Tipping, M. E., and Bishop, C. M. (1999), “Mixtures of Probabilistic Principal Component Analyzers,” *Neural Computation*, 11, 443–482.
- Vasicek, O. (1977), “An equilibrium characterization of the term structure,” *Journal of Financial Economics*, 5, 177–188.
- Wright, J. H. (2006), “The Yield Curve and Predicting Recessions,” *Finance and Economics Discussion Series* No. 2006-07, Federal Reserve Board, 2006.

Appendix 1: Alternative estimation methods

A.1 Unrestricted variances and the Kalman filter

In the more general case of the term structure model, e.g., as in Fisher and Gilles (1996) and De Jong (2000), each yield is assumed to be observed with error. The covariance matrix of these errors, R , is estimated as a set of parameters, possibly subject to identification restrictions. With this structure, maximum likelihood estimates of the covariance matrix of errors and of the

other parameters of the model are typically obtained by a straightforward but computationally intensive series of applications of the Kalman filter.

The Kalman filter is used to estimate x_t given the values of $a, b, M, R,$ and Q , which in turn may all be derived from Θ . As customary, let $x_{t|s}$ represent an estimate of x_t conditional on information available at time s and let $P_{t|s}$ be the conditional variance of the estimate. With observations for periods $t = 1, 2, \dots, T$, there is no contemporaneous information about x_0 . A standard assumption is that x_0 has the unconditional distribution of x_t , namely $N(0, V_x)$, and so

$$x_{0|0} = 0 \text{ and } P_{0|0} = V_x. \quad (31)$$

Estimates for $t = 1, 2, \dots, T$ are then obtained from the recursive Kalman equations

$$x_{t|t-1} = Mx_{t-1|t-1} \quad (32)$$

$$P_{t|t-1} = MP_{t-1|t-1}M' + Q \quad (33)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}b(b'P_{t|t-1}b + R)^{-1}b'P_{t|t-1} \quad (34)$$

$$x_{t|t} = x_{t|t-1} + P_{t|t-1}b(b'P_{t|t-1}b + R)^{-1}(y_t - a - b'x_{t|t-1}). \quad (35)$$

For any set of parameter values Θ , the likelihood function (14) is evaluated by setting

$E(x_t | y_{t-1}) = x_{t|t-1}$ and $V(x_t | y_{t-1}) = P_{t|t-1}$ for $t = 1, 2, \dots, T$ and is maximized with respect

to Θ . One drawback of this method is that optimization of the likelihood function with three or more state variables tends to be time consuming, numerically challenging, and very sensitive to starting values of the parameters.

A.2 Chen-Scott method

Chen and Scott (1993) proposed a simplified maximum likelihood approach that assumes that the yields for q maturities, the same number as there are state variables in the model, are

observed without error. Yields for the other $n-q$ maturities, if any, are observed with error and their variances are estimated as parameters. This method is computationally faster than the Kalman filter approach and may be seen as producing a constrained maximum likelihood estimate of the model.

Suppose that the elements of y_t are sorted so that the first q are observed without error. Analogously, let R be diagonal with $R_{i,i} = 0$ for $i = 1, \dots, q$. If $a_{(q)}$ and $b'_{(q)}$ are the submatrices of a and b' corresponding to the first q rows, and if $b_{(q)}$ is nonsingular, x_t may be obtained directly from the relationship

$$x_t = b_{(q)}^{-1} (y_t - a_{(q)}). \quad (36)$$

Moreover, $E(x_t | y_{t-1}) = Mx_{t-1}$ and $V(x_t | y_{t-1}) = Q$. These values may be used in the likelihood function (14), together with the condition that $E\varepsilon_t u_t' = 0$, to simplify the expression.

As in the Kalman filter approach, there is still the issue that there is no direct contemporaneous information about x_0 . One alternative, as before, is to suppose that x_0 has the unconditional distribution $N(0, V_x)$ hence $E(x_1 | y_0) = 0$ and $V(x_1 | y_0) = MV_x M' + Q = V_x$. A simpler alternative, for instance as in Ang and Piazzesi (2003), is to exclude the first observation altogether from the calculation of the likelihood function. In that case,

$$L(\Theta) = -\sum_{t=2}^T (2 \log \|b_{(q)}\| + \log |Q| + \varepsilon_t' Q^{-1} \varepsilon_t + \log |R| + u_t' R^{-1} u_t), \quad (37)$$

where parameters are evaluated at the estimated values.

With the Chen-Scott method, computational time is greatly reduced as compared with the Kalman filter, since equation (36) involves far fewer calculations. However, a well-known drawback of the method is that the choice of the particular maturities observed without error is ad hoc, and different choices lead in general to different results.

Appendix 2: Discrete versus continuous models

A significant portion of the term structure modeling literature has been based on continuous time models.²⁰ Dai, Le and Singleton (2006) show that discrete time models of the type used in the present paper converge to canonical Dai-Singleton (2000) and Duffee (2002) continuous time models as the length of the time period approaches zero. The converse equivalence, however, is less straightforward. An essentially affine ($\lambda_1 \neq 0$) continuous time model with transition equation

$$dx_t = K(\theta - x_t)dt + \Sigma dW_t \quad (38)$$

where W_t is a Wiener process, integrates to a standard discrete time model only if the price of risk is a step function of time such that

$$\Lambda_{t+s} = \lambda_0 + \lambda_1 x_t \quad (39)$$

for t an integer and s real with $0 \leq s < 1$.

²⁰ E.g., Vasicek (1977), Duffie and Kan (1996), DeJong (2000), Dai and Singleton (2000, 2002).

Table 1. Summary Statistics of Data

Fama-Bliss Jan. 1959 to Dec. 2007					
Maturity	Mean	Median	Std dev	Max	Min
1 Month	5.239	4.905	2.646	16.150	0.794
3 Month	5.568	5.132	2.774	16.042	0.898
1 Year	5.969	5.527	2.744	15.812	1.039
2 Year	6.168	5.734	2.684	15.639	1.299
3 Year	6.338	5.871	2.605	15.557	1.621
4 Year	6.469	6.010	2.562	15.824	1.998
5 Year	6.543	6.127	2.527	15.001	2.351
Federal Reserve Board Aug. 1971 to Dec. 2007					
Maturity	Mean	Median	Std dev	Max	Min
3 Month	6.110	5.546	3.022	16.378	0.904
6 Month	6.382	5.731	3.116	17.278	0.978
1 Year	6.497	6.060	2.943	16.110	1.030
2 Year	6.736	6.360	2.821	15.780	1.330
3 Year	6.908	6.545	2.727	15.570	1.640
4 Year	7.043	6.665	2.651	15.350	1.980
5 Year	7.156	6.800	2.588	15.180	2.330
6 Year	7.252	6.970	2.534	15.060	2.650
7 Year	7.336	7.085	2.487	14.990	2.950
8 Year	7.410	7.170	2.445	14.940	3.220
9 Year	7.475	7.235	2.409	14.910	3.460
10 Year	7.532	7.290	2.379	14.890	3.670

Table 2. Estimates with Fama-Bliss data, Jan. 1959 – Dec. 2007 (588 observations)

Parameter	Exact Principal Factors	Chen-Scott Method	Kalman Filter
M_{11}	0.986 (0.113)	0.987 (0.145)	0.989 (0.095)
M_{21}	0.033 (0.204)	0.033 (0.224)	0.027 (0.207)
M_{22}	0.943 (0.300)	0.945 (0.331)	0.968 (0.255)
M_{31}	0.050 (0.255)	0.038 (0.279)	0.024 (0.223)
M_{32}	-0.069 (0.379)	-0.079 (0.365)	-0.031 (0.340)
M_{33}	0.662 (0.605)	0.640 (0.586)	0.728 (0.641)
λ_{01}	-0.217 (1.120)	-0.209 (0.916)	-0.166 (1.742)
λ_{02}	-0.009 (0.824)	-0.027 (1.118)	-0.177 (2.192)
λ_{03}	-0.634 (3.887)	-0.594 (3.990)	-0.654 (4.090)
λ_{11}	-0.025 (0.127)	-0.022 (0.165)	-0.022 (0.109)
λ_{12}	-0.004 (0.110)	0.006 (0.090)	-0.006 (0.132)
λ_{13}	0.117 (0.103)	0.112 (0.119)	0.123 (0.180)
λ_{21}	0.019 (0.201)	0.010 (0.223)	-0.005 (0.209)
λ_{22}	-0.029 (0.299)	-0.012 (0.336)	0.014 (0.250)
λ_{23}	-0.006 (0.055)	0.023 (0.072)	0.043 (0.179)
λ_{31}	-0.101 (0.227)	-0.090 (0.221)	-0.091 (0.205)
λ_{32}	0.077 (0.388)	0.091 (0.378)	0.059 (0.342)
λ_{33}	0.144 (0.613)	0.098 (0.631)	0.175 (0.658)
δ_0	5.112 (29.001)	5.248 (32.258)	5.211 (26.686)
δ_{11}	0.426 (0.263)	0.442 (0.367)	0.436 (0.274)
δ_{12}	-0.045 (0.535)	-0.078 (0.572)	-0.118 (0.527)
δ_{13}	0.383 (0.265)	0.404 (0.267)	0.344 (0.324)
Log likelihood	4561.	799.	525.
Mean standard error	0.090	0.116	0.094

Note: standard errors in parentheses.

Table 3. Covariance matrix (R) of measurement errors, Fama-Bliss data, Jan. 1959 – Dec. 2007

Method\Maturity (yrs.)	.083	.25	1	2	3	4	5
Exact principal factors	0.008	0.008	0.008	0.008	0.008	0.008	0.008
Chen-Scott	0	0.059	0	0.014	0.012	0.010	0
Kalman filter	0.014	0.014	0.014	0.014	0.014	0.014	0.014

Note: Matrices are diagonal.

Table 4. Probit Regression with Fama-Bliss data (Jan. 1960 – Dec. 2005)

Model	(1)	(2)	(3)	(4)	(5)
Constant term	-0.605 (-2.92)	-0.651 (-3.17)	-1.29 (-6.20)	-0.910 (-3.57)	-0.787 (-2.79)
5-Year / 3-Month Spread		-0.780 (-4.34)			
5-Year / 3-Month Expectation Component			-0.593 (-3.68)		-0.835 (-3.93)
5-Year / 3-Month Term Premium Component				-0.190 (-0.94)	-0.633 (-3.15)
10-Year / 3-Month Treasury Spread (for reference)	-0.737 (-4.26)				
Number of observations	552	552	552	552	552
Log likelihood	-152.601	-159.130	-180.029	-216.082	-157.048
R square	0.251	0.225	0.145	0.012	0.233

Note: t-statistics in parentheses. HAC standard errors are computed using a 12-month Newey-West window. Cannot reject $\beta_1 = \beta_2$ in model (5).

Table 5. Probit Regression with Federal Reserve Board data (Aug. 1972 – Dec. 2005)

Model	(1)	(2)	(3)	(4)	(5)
Constant term	-0.557 (-2.35)	-0.622 (-2.71)	-1.666 (-7.04)	-0.839 (-1.41)	-0.581 (-1.11)
10-Year / 3-Month Spread		-0.711 (-4.27)			
10-Year / 3-Month Expectation Component			-0.546 (-4.38)		-0.711 (-4.29)
10-Year / 3-Month Term Premium Component				-0.144 (-0.45)	-0.736 (-2.52)
10-Year / 3-Month Treasury Spread (for reference)	-0.716 (-4.08)				
Number of observations	401	401	401	401	401
Log likelihood	-98.547	-94.026	-108.869	-157.414	-94.008
R square	0.313	0.338	0.257	0.005	0.338

Note: t-statistics in parentheses. HAC standard errors are computed using a 12-month Newey-West window. Cannot reject $\beta_1 = \beta_2$ in model (5).

Table 6. Probit Regression with Kim-Wright term premiums (Jul. 1991 – Dec. 2005)

Model	(1)	(2)	(3)	(4)	(5)
Constant term	-0.437 (-1.03)	-0.166 (-0.43)	-2.50 (-6.05)	-0.33 (-0.48)	-1.49 (-2.46)
10-Year / 3-Month Spread		-2.15 (-5.92)			
10-Year / 3-Month Expectation Component			-2.85 (-3.66)		-3.30 (-6.54)
10-Year / 3-Month Term Premium Component				-1.36 (-2.78)	-1.21 (-2.07)
10-Year / 3-Month Treasury Spread (for reference)	-2.28 (-5.04)				
Number of observations	174	174	174	174	174
Log likelihood	-13.746	-13.921	-13.633	-28.025	-11.150
R square	0.274	0.271	0.276	0.053	0.329

Note: t-statistics in parentheses. HAC standard errors are computed using a 12-month Newey-West window. Can reject $\beta_1 = \beta_2$ in model (5).

Figure 1. Term premiums: Fama-Bliss data from Jan. 1959 to Dec. 2007

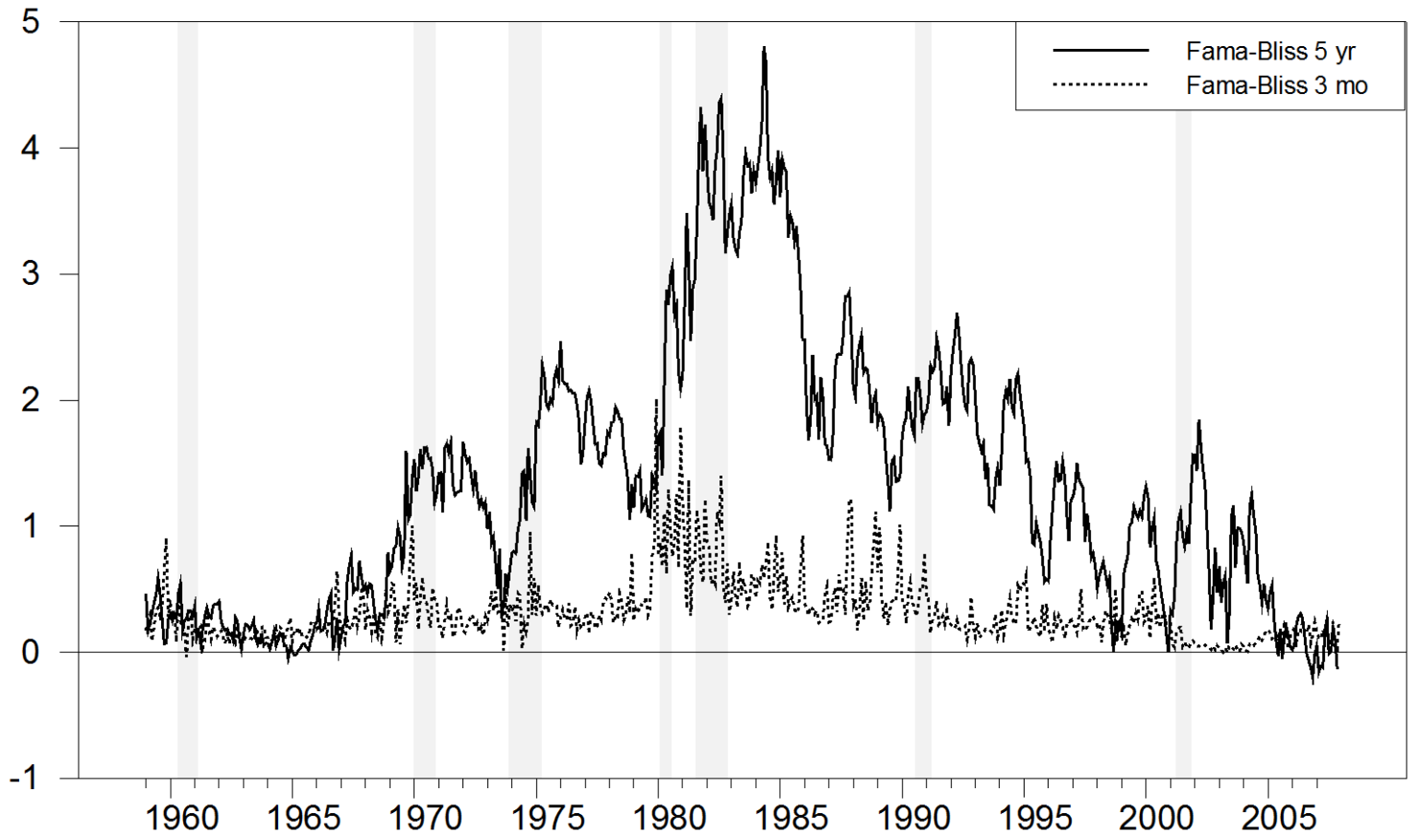


Figure 2. Long-term term premiums: comparisons from Jul. 1990 to Dec. 2007

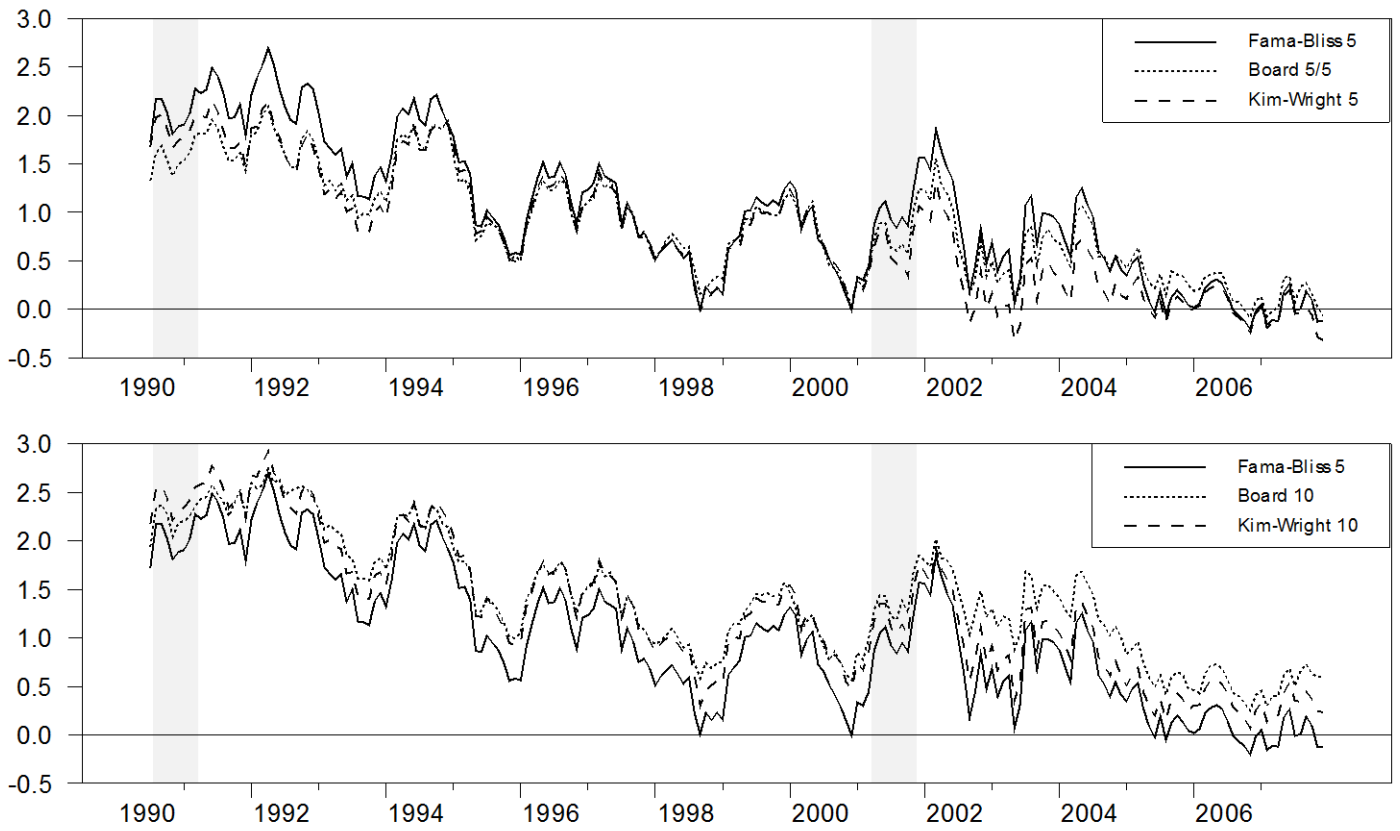


Figure 3. Autocorrelation of spread errors

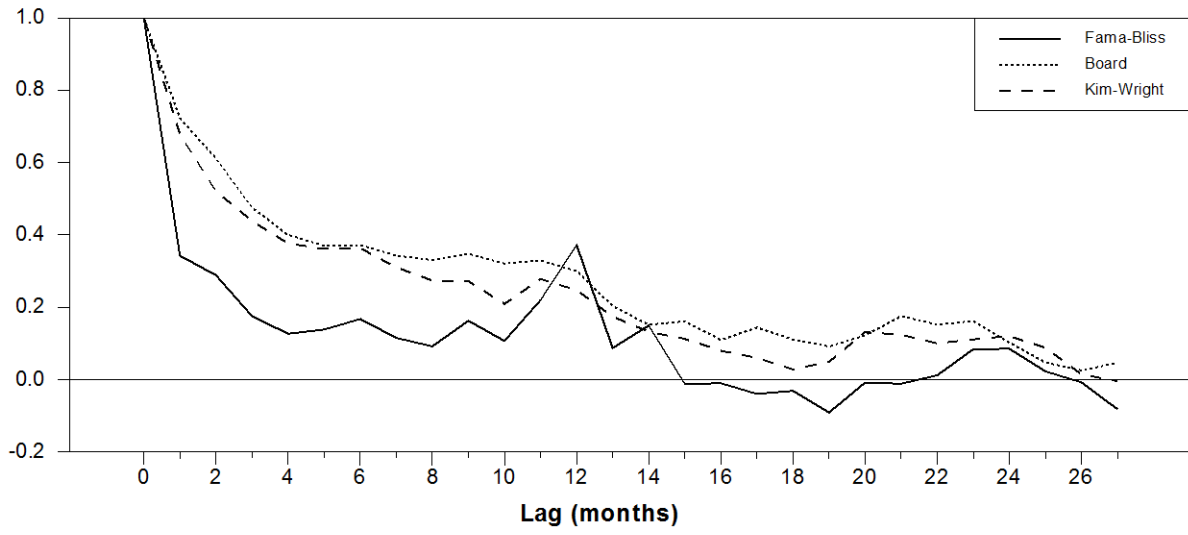


Figure 4. Campbell-Shiller tests: Fama-Bliss versus Board data

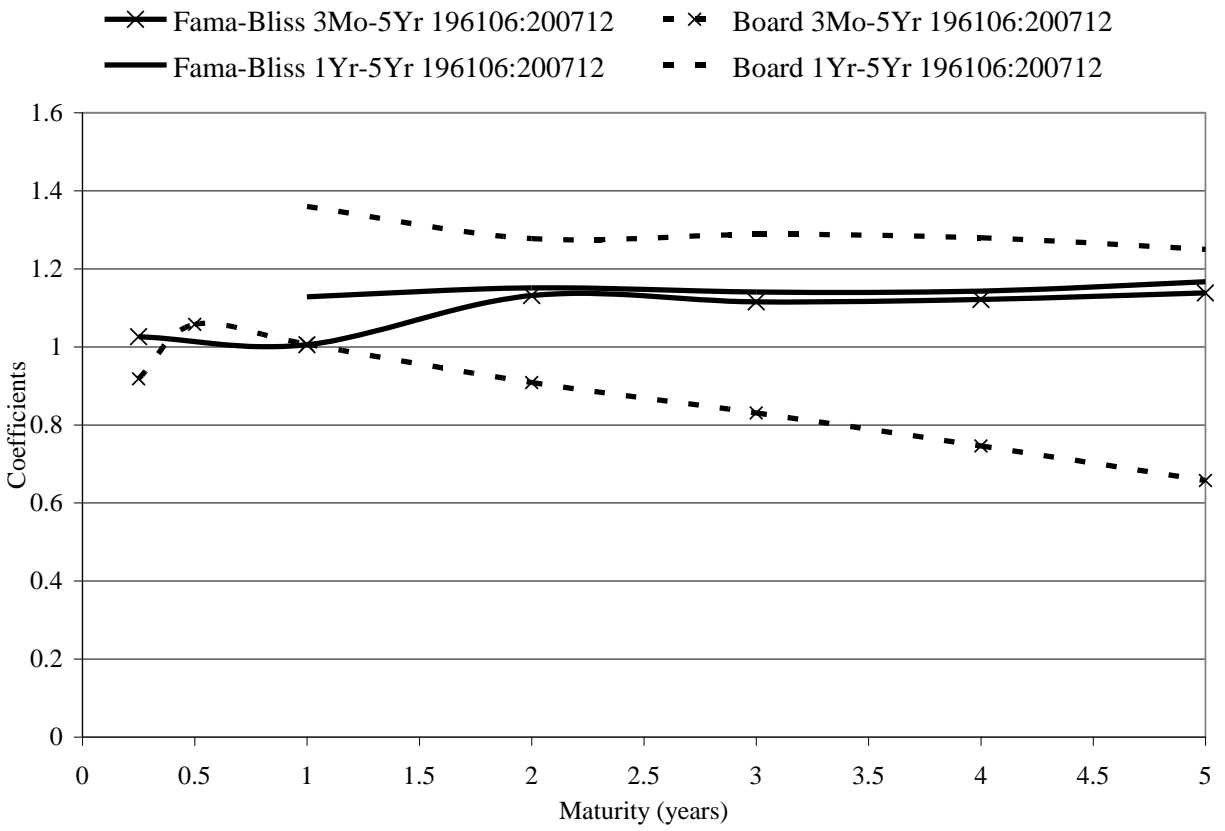


Figure 5. Campbell-Shiller tests: maturity and sample period effects with Board data

